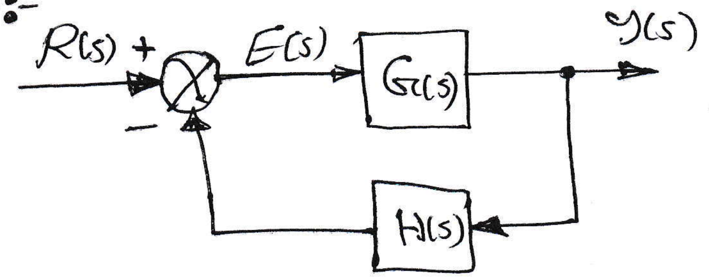
5-1 Steady State Errors :-

5-1-1 Error Constants :-

let us consider a feedback control system.



$$E(s) = R(s) - H(s)\,y(s)$$

$$y(s) = G(s)\,E(s)$$

$$E(s) = R(s) - H(s)\,G(s)\,E(s)$$

$$E(s)\left[1 + G(s)H(s)\right] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

The final value theorem, we can get the steady state error e_{ss} as

$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \frac{s\,R(s)}{1 + G(s)\,H(s)}$$

1- Unit step or position input :-

for a unit step input, $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s \to 0} s \frac{R(s)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \cancel{s} \frac{\frac{1}{\cancel{s}}}{1 + G(s)H(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)H(s)}$$

the "position error constant K_p" as

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$\therefore \quad e_{ss} = \frac{1}{1 + K_p}$$

2- Unit ramp or velocity input :-

For unit velocity input, $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \frac{\frac{1}{s^{\cancel{2}}}}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s(1 + G(s)H(s))}$$

$$= \lim_{s \to 0} \frac{1}{s + s\,G(s)H(s)}$$

$$= \frac{1}{\lim\limits_{s \to} s\, G(s) H(s)}$$

the "velocity error constant" K_v" as

$$K_v = \lim_{s \to 0} s\, G(s) H(s)$$

$$e_{ss} = \frac{1}{K_v}$$

3- Unit parabolic or acceleration input.

For unit acceleration input $R(s) = \frac{1}{s^3}$,

$$e_{ss} = \lim_{s \to 0} \frac{s}{s^3 [1 + G(s) H(s)]}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s) H(s)}$$

$$= \frac{1}{\lim\limits_{s \to 0} s^2 G(s) H(s)}$$

"the acceleration error constant K_a", as

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$e_{ss} = \frac{1}{K_a}$$

For the special case of unity of feedback system, $H(s)=1$

$$K_p = \lim_{s \to 0} G(s)$$

$$K_v = \lim_{s \to 0} s\,G(s)$$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

5-1-2 Dependence of steady state Error on Type of the System

1 - type - 0 system

$$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{K(T_{z1}s+1)(T_{z2}s+1)\cdots}{(T_{p1}s+1)(T_{p2}s+1)\cdots} = K$$

$$K_v = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} \frac{sK(T_{z1}s+1)(T_{z2}s+1)\cdots}{(T_{p1}s+1)(T_{p2}s+1)\cdots} = 0$$

$$K_a = \lim_{s \to 0} s^2 G(s) = 0$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} \quad \text{(step input)}$$

$$e_{ss} = \frac{1}{K_v} = \infty \quad \text{(velocity input)}$$

$$e_{ss} = \frac{1}{K_a} = \infty \quad \text{(acceleration input)}$$

2- Type 1 system

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K}{s} = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{K}{\cancel{s}} = K$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^2\cdot\frac{K}{s} = 0$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{\infty} = 0$$ (position)

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$ (velocity)

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$ (acceleration)

3- Type 2-system

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{K}{s^2} = \infty$$

$$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} \frac{sK}{s^2} = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \frac{s^2K}{s^2} = K$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$ (position)

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$ (velocity)

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K}$$ (acceleration)

Example Find the steady state error for unit step, unit ramp and unit acceleration inputs for the following systems.

1- $\dfrac{10}{s(0.1s+1)(0.5s+1)}$ 2- $\dfrac{1000(s+1)}{(s+10)(s+50)}$ 3- $\dfrac{1000}{s^2(s+1)(s+20)}$

(1) $G(s) = \dfrac{10}{s(0.1s+1)(0.5s+1)}$

(a) Unit step input

$$K_p = \lim_{s\to 0} G(s) = \frac{10}{0} = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{\infty} = 0$$

(b) Unit ramp input

$$K_v = \lim_{s\to 0} s\,G(s)$$

$$= \lim_{s\to 0} \cancel{s}\cdot\frac{K}{\cancel{s}} = 10$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

(c) Unit acceleration input

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^2\cdot\frac{K}{s} = 10s = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

2) $$G(s) = \frac{1000(s+1)}{(s+10)(s+50)}$$

$$G(s) = \frac{1000(s+1)}{10 \times 50\,(0.1s+1)(0.02s+1)}$$

$$= \frac{2(s+1)}{(0.1s+1)(0.02s+1)}$$

Unit step

① $$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{2(s+1)}{(0.1s+1)(0.02s+1)} = 2$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+2} = \frac{1}{3}$$

② Unit ramp

$$K_v = \lim_{s \to 0} s\,G(s) = \frac{2s(s+1)}{(0.1s+1)(0.02s+1)} = 0$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

③ Unit acceleration input

$$K_a = \lim_{s \to 0} s^2 G(s) = \frac{2s^2(s+1)}{(0.1s+1)(0.2s+1)} = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

③ $G(s) = \dfrac{1000}{s^2(s+1)(s+20)}$

$$= \frac{1000}{20s^2(s+1)(0.05s+1)}$$

$$G(s) = \frac{50}{s^2(s+1)(0.05s+1)}$$

ⓐ Unit step input

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{50}{s^2(s+1)(0.05s+1)}$$

$$= \infty$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

b) Unit ramp input

$$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} s\,\frac{50}{s^2(s+1)(0.05s+1)}$$

$$= 0$$

$$e_{ss} = \frac{1}{K_v} = \infty$$

c) Unit acceleration input

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} s^2 \frac{50}{s^2(s+1)(0.05s+1)}$$

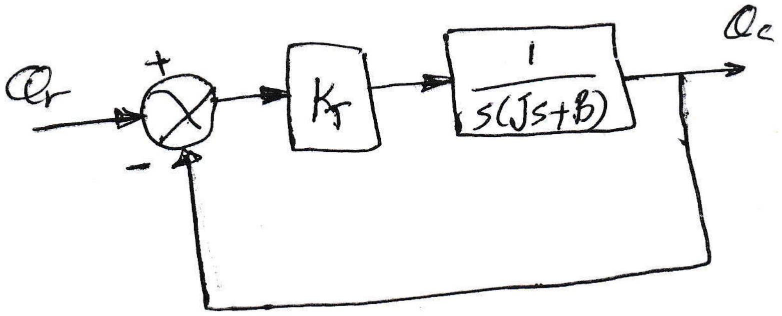
$$= 50$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{50} = 0.02$$

EX The angular position θ_c of a mass is controlled by a servo system through a reference signal θ_r. The moment of inertia of moving parts referred to the load shaft, J, is 150 Kgm² and damping torque coefficient referred to the load shaft, B, is 4.5×10^3 Nwm/rad/sec. The torque developed by the motor at the load is 7.2×10^4 Nw-m per radian of error.

a) obtain the response of the system to step input of 1 rad and determine the peak time, peak overshoot and frequency of transient oscillations. Also find the steady state error for a constant angular velocity of 1 revolution/minute.

b) If a steady torque of 1000 Nwm is applied at the load shaft, determine the steady state error.



Feedforward path transfer function $G(s) = \dfrac{K_T}{s(Js+B)}$

$$G(s) = \frac{7.2 \times 10^4}{s(150s + 4.5 \times 10^3)}$$

$$G(s) = \frac{16}{s(0.333s+1)}$$

$K = 16, \quad \tau = 0.333$ sec

$$\delta = \frac{1}{2\sqrt{K\tau}} = \frac{1}{2\sqrt{16 * 0.333}} = 0.6847$$

$$w_n = \sqrt{\frac{K}{\tau}} = \sqrt{\frac{16}{0.333}} = 21.91 \text{ rad/sec}$$

$$\theta(t) = 1 - \frac{e^{-\delta w_n t}}{\sqrt{1-\delta^2}} \sin\left(w_n\sqrt{1-\delta^2}\, t + \tan^{-1}\sqrt{\frac{1-\delta^2}{\delta}}\right)$$

$$= 1 - 1.372\, e^{-15t} \sin(15.97t + 46.8^\circ)$$

Peak time

$$t_p = \frac{\pi}{w_n\sqrt{1-\delta^2}} = \frac{\pi}{w_d} = \frac{\pi}{15.97} = 0.1967 \text{ sec}$$

Peak over shoot, $M_p = 100\, e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}$

$$= 5.23\ \%$$

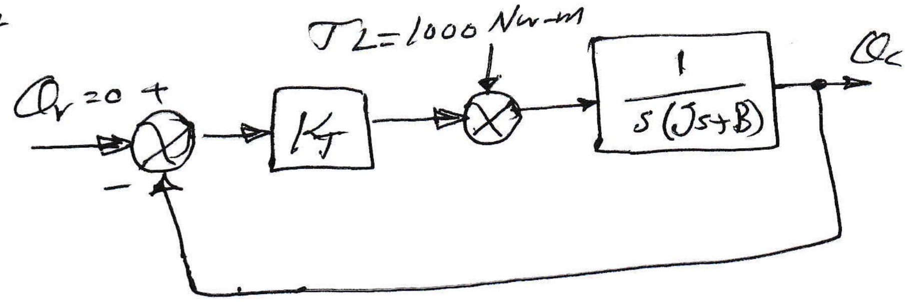
Frequency of transient ~~and~~ oscillations, $w_d = 15.97$ rad/sec

steady state error $\dot{\theta} = \frac{2\pi}{60}$ rad/sec

$$e_{ss} = \frac{2\pi}{60 * 16} = 6.54 \times 10^{-3} \text{ rad}$$

b) when a load torque of 1000 Nwm is applied at the load shaft, using super position theorem, the error is nothing but the response due to this load torque acting as a step input with

$\theta_r = 0$.



$$\frac{\theta_c(s)}{T_L(s)} = \frac{\frac{1}{s(Js+B)}}{1 + \frac{K_T}{s(Js+B)}} = \frac{1}{Js^2 + Bs + K_T}$$

$$\theta_c = \frac{1000}{s(150s^2 + 4.5 \times 10^3 s + 7.2 \times 10^4)}$$

final value theorem

$$\theta_{css} = \lim_{s \to 0} s\,\theta_c(s) = \frac{1000}{7.2 \times 10^4}$$

$$= 0.01389 \text{ rad}$$
$$= 0.796^\circ$$

Ex The open loop transfer function of a unity feedback system is given by,

$$G(s) = \frac{K}{s(Ts+1)} \quad , K, T > 0$$

with a given value of K, the peak overshoot was found to be 80%. It is proposed to reduce the peak overshoot to 20% by decreasing the gain. Find the new value of K in terms of the old value.

let the gain be K_1 for a peak overshoot of 80%.

$$e^{-\frac{\pi \delta_1}{\sqrt{1-\delta_1^2}}} = 0.8$$

$$\frac{\pi \delta_1}{\sqrt{1-\delta_1^2}} = \ln \frac{1}{0.8} = 0.223$$

$$\pi \delta_1 = 0.223\sqrt{1-\delta_1^2}$$

$$\pi^2 \delta_1^2 = 0.223^2(1-\delta_1^2)$$

$$\therefore \pi^1 \delta_1^2 + 0.223^2 \delta_1^2 = 0.223^2$$

$$\Rightarrow \delta_1 = 0.07$$

let the new gain be K_2 for a peak overshoot of 20%

$$e^{\frac{-\pi \delta_2}{\sqrt{1-\delta_2^2}}} = 0.2$$

$$\frac{\pi \delta_2}{\sqrt{1-\delta_2^2}} = \ln 0.2 \Rightarrow \frac{\pi \delta_2}{\sqrt{1-\delta_2^2}} = 1.61$$

the same way to find δ_2,

$$\delta_2 = 0.456$$

$$\delta = \frac{1}{2\sqrt{KT}}$$

$$\frac{\delta_1}{\delta_2} = \frac{2\sqrt{K_2 T}}{2\sqrt{K_1 T}} = \sqrt{\frac{K_2}{K_1}}$$

$$\frac{\delta_1^2}{\delta_2^2} = \frac{K_2}{K_1} \Rightarrow K_2 = \frac{\delta_1^2}{\delta_2^2} \cdot K_1 = 0.0236\, K_1$$

Ex Find t_r, t_s and M_p for the system

$$T.F. = \frac{30(s-6)}{s(s^2+4s+13)}$$

① $t_r = \dfrac{1.8}{w_n}$

$$s^2+4s+13 = s^2 + 2\zeta w_n s + w_n^2$$

$$\therefore w_n^2 = 13 \Rightarrow w_n = \sqrt{13}$$

$$\therefore t_r = \frac{1.8}{\sqrt{13}} = 0.5 \text{ sec}$$

② $t_s = \dfrac{4}{\zeta w_n}$

$$4s = 2\zeta w_n s \Rightarrow 4 = 2\zeta w_n$$

$$\zeta = \frac{4}{2*\sqrt{13}} = \frac{4}{2*3.6} = \frac{4}{7.211} = 0.55$$

$$\therefore t_s = \frac{4}{0.55*\sqrt{13}} = \frac{4}{1.98} = 2.02 \text{ sec}$$

③ $M_p = 100\, e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow M_p = 100\, e^{-\frac{1.727}{0.835}}$

$M_p = 12.64$

EX The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s+2)}$$

The desired system response to a step input is specified as peak time $t_p = 1$ sec, and overshoot $M_p = 5\%$.

a) Determine wheather both specifications can be met simultaneously by selecting the right value of K.

b)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K}{s^2+2s+K} = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

$$\omega_n^2 = K \Rightarrow \omega_n = \sqrt{K}$$

$$2s = 2\delta\omega_n s \Rightarrow \delta = \frac{1}{\omega_n} \Rightarrow \delta = \frac{1}{\sqrt{K}}$$

$$M_p = 100\, e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \frac{5}{100} = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \Rightarrow \delta = 0.69$$

$$t_p = \frac{\pi}{\omega_d} \Rightarrow 1\text{ sec} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} \Rightarrow \omega_n = 4.34$$

the combination $\delta = 0.69$, $\omega_n = 4.34$ is not possible by varying K, because we have in the begining

$$\delta\omega_n = \sqrt{K}\cdot\frac{1}{\sqrt{K}} = 1$$

but

$$\delta\omega_n = 0.69 * 4.34 = 2.99$$

is not the same.

EX Find the allowable regions in the S-plane for the poles of a transfer function of a system if the system response requirements are $t_r \le 0.6$ sec, $M_p \le 10\%$, and $t_s \le 3$ sec

$$t_r = \frac{1.8}{w_n} \Rightarrow w_n = \frac{1.8}{0.6} = 3 \text{ rad/sec.}$$

$$M_p = 100\, e^{-\frac{\pi \delta}{\sqrt{1-\delta^2}}} \quad \text{or} \quad \delta = \frac{\ln(M_p)}{\sqrt{\pi^2 + (\ln(M_p))^2}}$$

$$\delta = \frac{\ln(10)}{\sqrt{\pi^2 + (\ln(10))^2}} \Rightarrow \delta = \frac{2.3}{\sqrt{3.14^2 + (2.3)^2}}$$

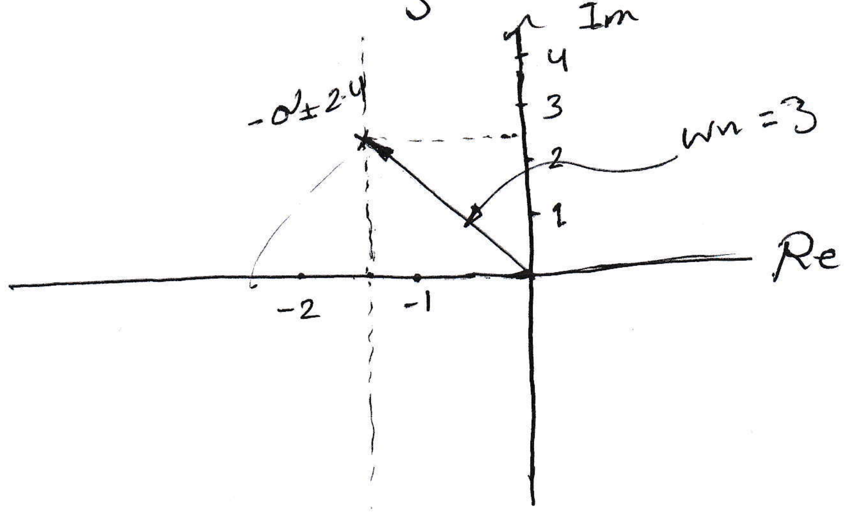
$$\delta \ge 0.6$$

$$t_s = \frac{4}{\delta w_n} \Rightarrow \delta w_n = \sigma = \frac{4}{3} = 1.3 \text{ sec.}$$

$$w_d = w_n \sqrt{1-\delta^2}$$

$$w_d = 3\sqrt{1-0.6^2}$$

$$w_d = 2.4$$



region of S-plane

EX For a second-order system with transfer function

$$G(s) = \frac{3}{s^2+2s-3}$$

determine the following:

a) the DC gain;
b) The final value to a step input.

a) DC gain $\Rightarrow$ $G(0) = \frac{3}{0+0-3} = -1$

b)

$\lim_{s \to 0} G(s)$ $\Rightarrow$ it is not applicable because the system is unstable

unstable.

$s^2+2s-3=0 \Rightarrow s = 1, -3$

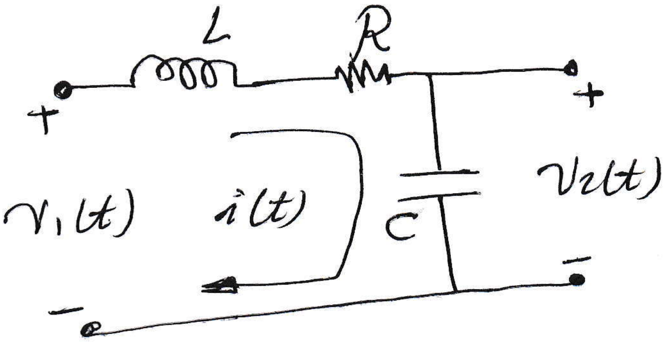
EX For the electric circuit shown in figure below, find

a) the time-domain equation relating $i(t)$ and $v_1(t)$:

b) the time-domain equation relating $i(t)$ and $v_2(t)$

c) Assuming all initial conditions are zero, the transfer function $V_2(s)/V_1(s)$ and the damping ratio δ and undamped natural frequency ω_n of the system:

d) the values of R that will result in $v_2(t)$ having an overshoot of no more than 25%, assuming $v_1(t)$ is a unit step, $L = 10\,mH$, and $C = 4\,\mu F$.



a) $v_1(t) = L\dfrac{di}{dt} + Ri + \dfrac{1}{C}\int i\,dt$

b) $v_2(t) = \dfrac{1}{C}\int i\,dt$

c) $\dfrac{V_2(s)}{V_1(s)} = \dfrac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \dfrac{1}{LCs^2 + RCs + 1}$

d) for 25% of M_p $\Rightarrow$ $\delta = \dfrac{\ln M_p}{\sqrt{\pi^2 + (\ln M_p)^2}} = 0.4$

$T.F = \dfrac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \Rightarrow \omega_n^2 = \dfrac{1}{LC} \Rightarrow \omega_n = \dfrac{1}{\sqrt{LC}}$

$2\delta\omega_n s = \dfrac{R}{L} \Rightarrow R = 2\delta\sqrt{\dfrac{L}{C}} = 2(0.4)\sqrt{\dfrac{10\times10^{-3}}{4\times10^{-6}}} = 40\,\Omega$

EX The equations of motion for the DC motor shown below,

$$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} V_a .$$

Assuming $J_m = 0.01\ Kg.m^2$

$b = 0.001\ N.m\text{-}sec$

$K_e = 0.02\ V.sec$

$K_t = 0.02\ N.m/A,$

$R_a = 10\ \Omega.$

a) Find the transfer function between the applied Voltage V_a and the motor speed $\dot{\theta}_m$.

b) What is the steady-state speed of the motor after a voltage $V_a = 10\ V$ has been applied?

c) Find the transfer function between the applied voltage V_a and the shaft angle θ_m.

d) if $V_a = K(\theta_r - \theta_m)$, for feedback is added, K is the feedback gain.

find ① K
② the transfer function between θ_r and θ_m

e) What is the maximum value of K that can be used if an overshoot $M_p < 20\%$?

f) What values of K will provide a rise-time of less than 4 sec?

ⓐ $$J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} V_a$$

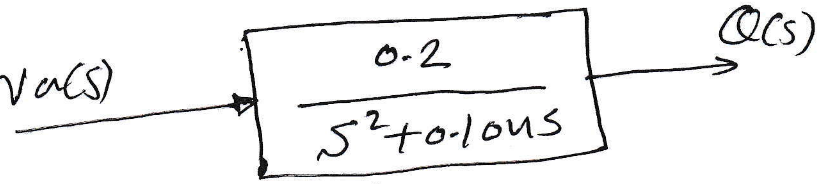
$$J_m S^2 \theta_m(s) + \left(b + \frac{K_t K_e}{R_a}\right) S\, \theta_m(s) = \frac{K_t}{R_a} V_a(s)$$

$$\therefore\ \theta_m(s)\left[J_m S^2 + \left(b + \frac{K_t K_e}{R_a}\right) S\right] = \frac{K_t}{R_a} V_a(s)$$

$$\therefore\ \frac{\theta(s)}{V_a(s)} = \frac{\frac{K_t}{R_a}}{J_m S^2 + \left(b + \frac{K_t K_e}{R_a}\right) S}$$

$$\frac{\theta(s)}{V_a(s)} = \frac{0.2}{S^2 + 0.104 S} = \frac{0.2}{S(S + 0.104)}$$

$$\frac{\theta(s) \cdot S}{V_a(s)} = \frac{0.2}{S + 0.104}$$



b) Final value theore to find $\dot{\theta}_{ss}$

we have $\theta(s)$, it is displacement but when we need the speed we must find the derivative of the displacement.

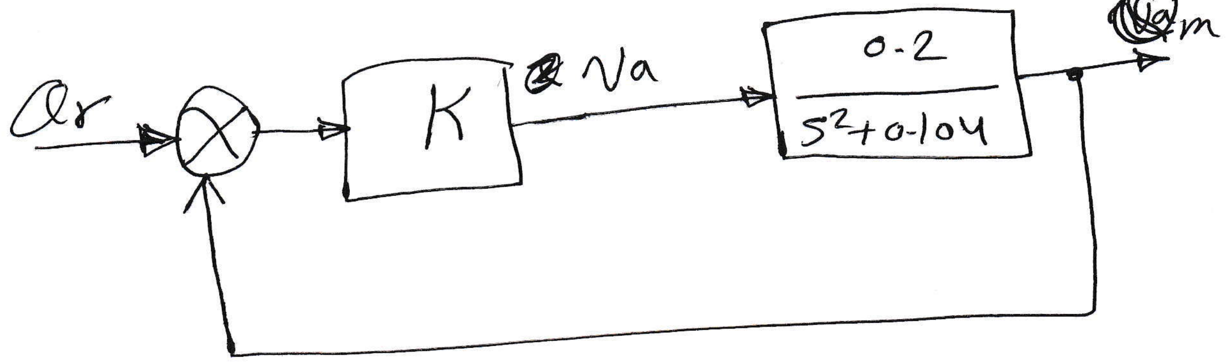
$$\frac{\theta(s)}{V_a(s)} = \frac{0.2}{S^2 + 0.104 S} \Rightarrow \theta(s) \cdot S = \frac{0.2 * V_a(s)}{S + 0.104}$$

$$\dot{\theta}(s) = \frac{0.2 * 10}{S(S + 0.104)}$$

$$\dot{\theta}(\infty) = \lim_{s \to 0} s\,\dot{\theta}(s) = s\,\frac{(0.2) * 10}{s(s+0.104)} = 19.23$$

c) $$\frac{\theta_m(s)}{V_a(s)} = \frac{0.2}{s(s+0.104)}$$

d) $$V_a = K(\theta_r - \theta_m)$$



$$\therefore \frac{\theta_m(s)}{\theta_r(s)} = \frac{\dfrac{0.2K}{s^2+0.104s}}{1+\dfrac{0.2K}{s^2+0.104s}} = \frac{0.2K}{s^2+0.104s+0.2K}$$

e) $$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} = 0.2$$

or $$\delta = \frac{\ln(M_p)}{\sqrt{\pi^2+(\ln(M_p))^2}} \Rightarrow \delta = 0.4559$$

$$\frac{0.2K}{s^2+0.104s+0.2K} = \frac{w_n^2}{s^2+2\delta w_n s+w_n^2}$$

$$2\delta w_n s = 0.104s \Rightarrow w_n = \frac{0.104}{2 * 0.4559} = 0.114 \text{ rad/sec}$$

$$w_n^2 = 0.2K \Rightarrow K < 6.5 * 10^{-2}$$

f) $t_r \geq \frac{1.8}{w_n}$

$$w_n \geq \frac{1.8}{t_r}$$

$$\sqrt{0.2K} \geq \frac{1.8}{t_r} \Rightarrow \sqrt{K} * 0.447 \geq 0.45$$

$$\sqrt{K} \geq \frac{0.45}{0.447} \Rightarrow \sqrt{K} = 1.0135$$

$$K \geq 1.01$$

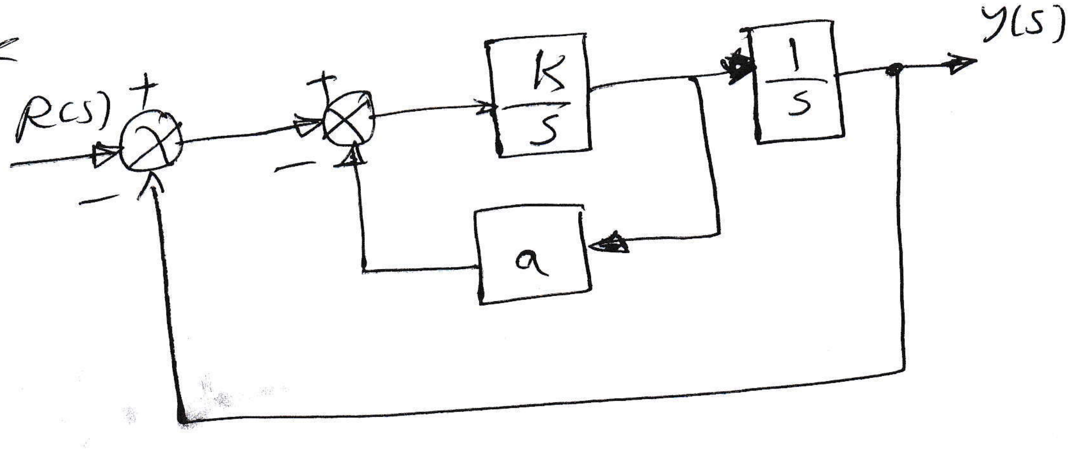
Time Domain Response Homework3

Q1 The open loop transfer function of a unity feedback Control system is given by,

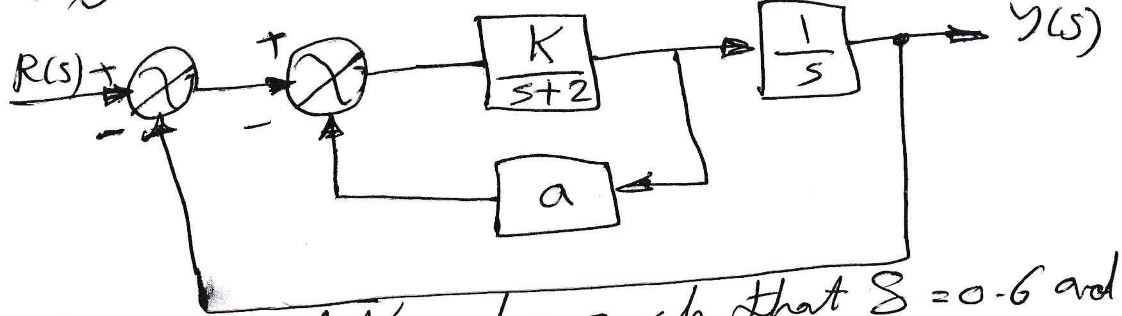
$$G(s) = \frac{K}{s(Ts+1)}$$

If the maximum response is obtained at $t = 4$ sec and the maximum value is 1.26, find the values K and T.

Q2 Find the values of K and a so that the Peak overshoot for a step input is 25% and Peak time is 2 sec



Q3 ⓐ Determine the values of K and a such that the damping factor is 0.6 and a settling time of 1.67 sec



b) Find the value of K and a such that $\delta = 0.6$ and e_{ss} for step input is 0.25. (102)

Q4 if you have $J\ddot{\theta} + B\dot{\theta} = T_c$ for sattelite-tracking antenna.

where T_c is the torque from the drive motor.

J = moment of inertia = 600,000 Kg.m²

B = is a damping = 20,000 N.m.sec.

a) Find the transfer function between applied torque T_c and the antenna angle θ.

b) if you have this relationship $T_c = K(\theta_r - \theta)$, where θ_r is a tracking a reference command, and K is the feedback gain. Find the transfer function between θ_r and θ.

c) what ~~values of~~ is the maximum value of K that can be used if you have an overshoot $M_p < 10\%$?

d) what values of K will provide a rise time of less than 80 sec?